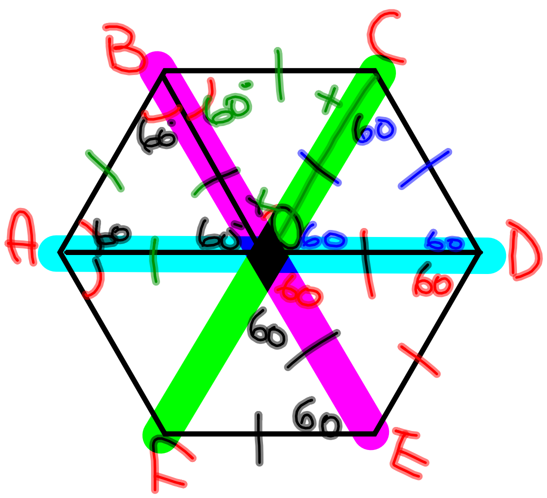
**9.3.7★** In this section we assumed that the long diagonals of a regular hexagon are concurrent. In this problem we fix this oversight by proving that these diagonals are concurrent.

(a) Let the hexagon be $ABCDEF$ and let point $O$ be the intersection of the bisectors of $\angle A$ and $\angle B$. Prove that $\triangle AOB$ is equilateral.

(b) Draw $\overline{OC}$. Prove that $\triangle BOC$ is equilateral.

(c) Prove that $\triangle COD$ is equilateral and that $\overrightarrow{AO}$ goes through $D$.

(d) Prove that the long diagonals of $ABCDEF$ all meet at the same point.




In a regular hexagon, all ∢'s are ≅, So, ∢B ≅ ∢A. Since BE bisects ∢B & AD bisects ∢A, & ∢A ≅ ∢B, ∢ABO ≅ ∢BAO b/c halves of ≅ parts are ≅.

∴ Since base ∢'s are ≅, ΔABO is isos Δ.

∢ by $\frac{180(n-2)}{n} \rightarrow 120$

**We know that BO=AB from part a. We also know that AB = BC, because it is a regular hexagon. By substitution, we now know that BO = BC. So, we get Triangle BCO is isosceles with vertex angle of 60. Since base angles are congruent, 60 + x + x = 180. Then, we get x = 60 as well. Therefore, Triangle BOC is also equilateral.**

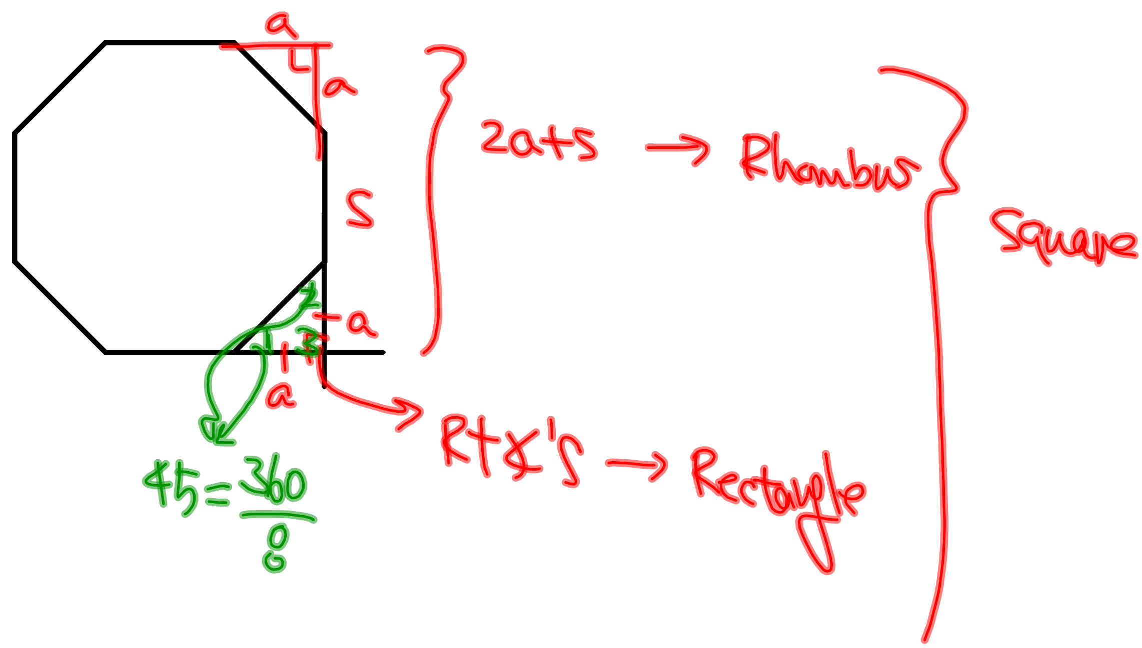
C) by the same reasoning as part b, we get Triangle COD is equilateral. Since angle AOB + angle BOC + angle COD = 180, we know that AOD is a straight line. This shows that AO when extended goes through point D.

d) ........

Since all three diagonals share point O, we know that the diagonals intersect at point O.

**December 21, 2015**

**9.3.6** We solved Problem 9.8 by extending the sides of a regular octagon to form a square. We didn't, however, prove that we form a square when we connect the points where these extensions meet. Fix this oversight by providing the proof. **Hints:** 564




Angle 1 and 2 are exterior angles of a regular octagon. Therefore their measures are 45 degrees by 360/8. Since two interior angles of a triangle are 45, we know that the third angle has to be 90 degrees in the triangle. This shows that the triangle is an isos. right triangle. when this is repeated at each corner, we get 4 right angles. This shows that the quadrilateral is a rectangle. If you let a side of the triangle as a and a side of the octagon as s, we know that the side of the quadrilateral is 2a + s. Then, we can say that it is a rhombus as well.

Since it is a rectangle and a rhombus, the quadrilateral has to be a square.

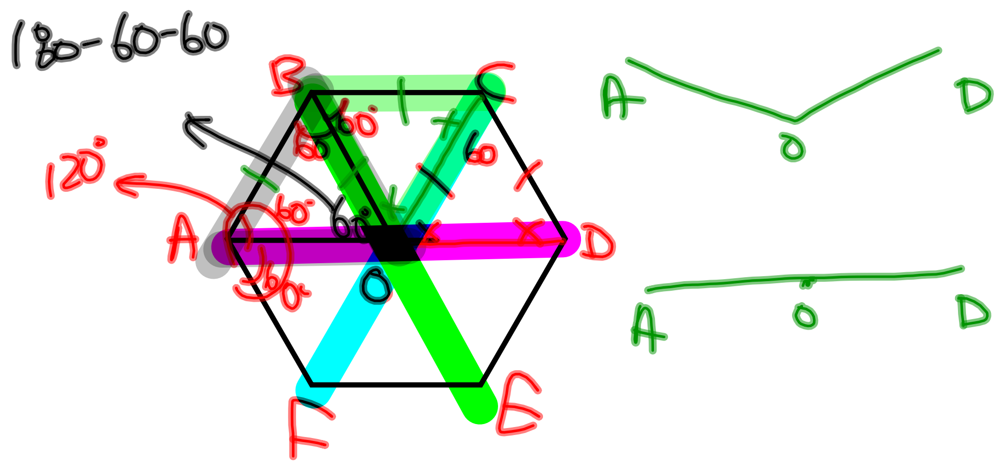
**9.3.7★** In this section we assumed that the long diagonals of a regular hexagon are concurrent. In this problem we fix this oversight by proving that these diagonals are concurrent.

(a) Let the hexagon be $ABCDEF$ and let point $O$ be the intersection of the bisectors of $\angle A$ and $\angle B$. Prove that $\triangle AOB$ is equilateral.

(b) Draw $\overline{OC}$. Prove that $\triangle BOC$ is equilateral.

(c) Prove that $\triangle COD$ is equilateral and that $\overrightarrow{AO}$ goes through $D$.

(d) Prove that the long diagonals of $ABCDEF$ all meet at the same point.




a) We know that the measure of an int. angle of a regular hexagon is 120. Since angle bisectors are drawn, angle OAB and OBA are 60 each. Then, we can figure out the third angle by using sum of int. angles of a triangle being 180. So, we know that all angles are 60 degrees, therefore the triangle is equilateral.

b) from part a, we know that AB = BO and since it is a regular hexagon, we also know that AB = BC. By substitution, BO = BC. We knew from part a that angle OBC was 60 due to an angle bisector. So, we have an isos. triangle with vertex angle measuring 60. This leads to 60 + x + x = 180. Then, we know that x = 60. So, we have a triangle with all interior angles of 60. This proves that the triangle is equilateral as well.

c) same reasoning as part b, we know that triangle COD is equilateral.

Then, angle AOB + angle BOC + angle COD = 180 because they are all measuring 60 degrees. So angle AOD is 180, this shows that when AO is extended it passes through point D.

d) using the reasoning from part a, b, and c, we know that triangle DOE is equilateral and BOE is a straight line as well. This applies to triangle EOF and line seg COF. Since three diagonals all share point O, we know that they all intersect at point O.

December 21, 2015